AMENDMENTS TO THE CLAIMS:

Claims 1-25 were pending at the time of the Office Action.

Claims 2-5, 8, 13-14, 19, and 21-24 are amended.

Claims 1-25 remain pending.

1. (Original) A method of model reduction and system identification of a dynamic system with multiple inputs, comprising:

generating a plurality of statistically independent random numbers for use as input signals; and

performing a singular-value-decomposition directly on a system response of the dynamic system due to a simultaneous excitation of the plurality of input signals.

2. (Currently Amended) The method of Claim 1, further comprising sampling individual pulse responses for a first time step and a second time step generated by subjecting the dynamic system to first and second pulses to provide Markov parameters for the first and second time steps.

3. (Currently Amended) The method of Claim 1, further comprising, for the system response generated by the simultaneous excitation of the plurality of input signals, sampling the [[a]] system response y^n for $n = 0, 1, 2, \ldots$ M to provide a Single-Composite-Input (SCI) response for a set of M steps.

4. (Currently Amended) The method of Claim 3 [[1]], further comprising defining Hankel-like matrices $\mathbf{H}_{c0}$ and $\mathbf{H}_{c1}$ using the SCI response for the set of M steps as follows:

$$\begin{aligned}\mathbf{H}_{c0} &\equiv [\mathbf{y}_{c0}^{1}\ \mathbf{y}_{c0}^{2}\ \cdots\ \mathbf{y}_{c0}^{M-1}] \\ &= \mathbf{C}[\mathbf{x}^{1}\ \mathbf{x}^{2}\ \ldots\ \mathbf{x}^{M-1}] \qquad (25)\\ \mathbf{H}_{c1} &\equiv [\mathbf{y}_{c1}^{1}\ \mathbf{y}_{c1}^{2}\ \cdots\ \mathbf{y}_{c1}^{M-1}] \\ &= \mathbf{C}\mathbf{A}[\mathbf{x}^{1}\ \mathbf{x}^{2}\ \ldots\ \mathbf{x}^{M-1}] \qquad (26)\end{aligned}$$

SVD of $\mathbf{H}_{c0}$ yields

$$\begin{aligned}\mathbf{H}_{c0} &\equiv \mathbf{U}\Sigma\mathbf{V}^{T} \\ &\simeq [\mathbf{U}_{R}\ \mathbf{U}_{D}]\begin{bmatrix}\Sigma_{R} & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{V}_{R}^{T}\\ \mathbf{V}_{D}^{T}\end{bmatrix} \\ &= \mathbf{U}_{R}\Sigma_{R}^{1/2}\Sigma_{R}^{1/2}\mathbf{V}_{R}^{T} \qquad (27)\end{aligned}$$

5. (Currently Amended) The method of Claim 4, ~~further comprising obtaining~~ wherein performing a singular-value decomposition includes performing a singular-value decomposition on the Hankel-like matrices $\mathbf{H}_{c0}$ and $\mathbf{H}_{c1}$ to obtain system matrices (A, B, C, D) ~~by a least square approximation~~ as follows:

$$\begin{aligned}\mathbf{D} &= \mathbf{Y}^{0} \qquad (28)\\ \mathbf{C} &\simeq \mathbf{U}_{R}\Sigma_{R}^{1/2} \qquad (29)\\ \mathbf{B} &\simeq \Sigma_{R}^{-1/2}\mathbf{U}_{R}^{T}\mathbf{Y}^{1} \qquad (30)\\ \mathbf{A} &\simeq \Sigma_{R}^{-1/2}\mathbf{U}_{R}^{T}\mathbf{H}_{c1}\mathbf{V}_{R}\Sigma_{R}^{-1/2} \qquad (31)\end{aligned}$$

6. (Original) The method of Claim 4, wherein $(M - 1) \geq R$ and $N0 \geq R$.

7. (Original) The method of Claim 4, wherein a total number of input samples is equal to $M + 1 + 2 \times Ni$.

8. (Currently Amended) The method of Claim <u>3</u> [[1]], further comprising defining augmented $\mathbf{H}_{c01}$ and $\mathbf{H}_{c11}$ matrices <u>using the SCI response for the set of M steps</u> as follows:

$$
\begin{aligned}
H_{c01} &\equiv \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^K \end{bmatrix} [x^1 \; x^2 \; \ldots \; x^{M-1}] \\
&= \begin{bmatrix} y_{c0}^1 & y_{c0}^2 & \cdots & y_{c0}^{M-1} \\ y_{c1}^1 & y_{c1}^2 & \cdots & y_{c1}^{M-1} \\ \cdots & \cdots & \cdots & \cdots \\ y_{cK}^1 & y_{cK}^2 & \cdots & y_{cK}^{M-1} \end{bmatrix} \qquad (32)
\end{aligned}
$$

$$
\begin{aligned}
H_{c11} &\equiv \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^K \end{bmatrix} A\,[x^1 \; x^2 \; \ldots \; x^{M-1}] \\
&= \begin{bmatrix} y_{c1}^1 & y_{c1}^2 & \cdots & y_{c1}^{M-1} \\ y_{c2}^1 & y_{c2}^2 & \cdots & y_{c2}^{M-1} \\ \cdots & \cdots & \cdots & \cdots \\ y_{cK+1}^1 & y_{cK+1}^2 & \cdots & y_{cK+1}^{M-1} \end{bmatrix} \qquad (33)
\end{aligned}
$$

where

$$
\begin{aligned}
y_{ck}^m &\equiv CA^k x^m \\
&= y^{m+k} - \sum_{i=1}^{N_i} y_i^0 r_i^{m+k} - \sum_{i=1}^{N_i} y_i^1 r_i^{m+k-1} - \\
&\quad \ldots - \sum_{i=1}^{N_i} y_i^k r_i^m
\end{aligned}
$$

9. (Original) The method of Claim 8, wherein a total number of input samples is equal to M +1+ K + (2+ K) x Ni.

10. (Original) The method of Claim 1, wherein at least some of the input signals are filtered through a low-pass filter.

11. (Original) The method of Claim 1, wherein the plurality of input signals includes applying multiple step inputs in a sequential manner.

12. (Original) The method of Claim 1, wherein the plurality of input signals includes applying multiple pulse inputs in a sequential manner.

13. (Currently Amended) The method of Claim 5 [[1]], further comprising performing a second order reduction on the system matrices (A, B, C, D) based on a ~~the~~ Frequency-Domain Karhunen-Loeve (FDKL) method ~~to the SCI/ERA-ROM using the plurality of input signals~~.

14. (Currently Amended) The method of Claim 13, further comprising premultiplying the system ~~SCI/ERA-ROM~~ matrices (A, B, C, D) by ΦT to yield a new reduced-order model as follows:

$$p^{n+1} = A_1 p^n + B_1 u^n \quad (53)$$

$$y^n = C_1 p^n + D u^n \quad (54)$$

where

$$A_1 \equiv \Phi^T A \Phi \quad (55)$$

$$B_1 \equiv \Phi^T B \quad (56)$$

$$C_1 \equiv C \Phi \quad (57)$$

15. (Original) A method of model reduction and system identification of a dynamic system with multiple inputs, comprising:

generating a plurality of statistically independent random numbers for use as input signals; and

performing a singular-value-decomposition directly on a system response of the dynamic system due to a simultaneous excitation of the plurality of input signals;

sampling individual pulse responses for a first time step and a second time step;

defining Hc0 and Hc1 matrices as follows:

$$
\begin{aligned}
\mathbf{H}_{c0} &\equiv \left[\mathbf{y}^1_{c0}\ \mathbf{y}^2_{c0} \cdots \mathbf{y}^{M-1}_{c0}\right] \\
&= \mathbf{C}\left[\mathbf{x}^1\ \mathbf{x}^2 \ldots \mathbf{x}^{M-1}\right] \qquad (25) \\
\mathbf{H}_{c1} &\equiv \left[\mathbf{y}^1_{c1}\ \mathbf{y}^2_{c1} \cdots \mathbf{y}^{M-1}_{c1}\right] \\
&= \mathbf{C}\mathbf{A}\left[\mathbf{x}^1\ \mathbf{x}^2 \ldots \mathbf{x}^{M-1}\right] \qquad (26)
\end{aligned}
$$

SVD of $\mathbf{H}_{c0}$ yields

$$
\begin{aligned}
\mathbf{H}_{c0} &\equiv \mathbf{U}\Sigma\mathbf{V}^T \\
&\simeq \left[\mathbf{U}_R\ \mathbf{U}_D\right]\begin{bmatrix}\Sigma_R & 0 \\ 0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{V}_R^T \\ \mathbf{V}_D^T\end{bmatrix} \\
&= \mathbf{U}_R\Sigma_R^{1/2}\Sigma_R^{1/2}\mathbf{V}_R^T \qquad (27)
\end{aligned}
$$

; and

obtaining system matrices (A, B, C, D) by a least square approximation as follows:

$$
\begin{aligned}
\mathbf{D} &= \mathbf{Y}^0 && (28) \\
\mathbf{C} &\simeq \mathbf{U}_R\Sigma_R^{1/2} && (29) \\
\mathbf{B} &\simeq \Sigma_R^{-1/2}\mathbf{U}_R^T\mathbf{Y}^1 && (30) \\
\mathbf{A} &\simeq \Sigma_R^{-1/2}\mathbf{U}_R^T\mathbf{H}_{c1}\mathbf{V}_R\Sigma_R^{-1/2} && (31)
\end{aligned}
$$

16. (Original) The method of Claim 15, wherein at least some of the input signals are filtered through a low-pass filter.

17. (Original) The method of Claim 15, wherein the plurality of input signals includes applying multiple step inputs in a sequential manner.

18. (Original) The method of Claim 15, wherein the plurality of input signals includes applying multiple pulse inputs in a sequential manner.

19. (Currently Amended) The method of Claim 15, further comprising performing a second order reduction on the system matrices (A, B, C, D) using a ~~the~~ Frequency-Domain Karhunen-Loeve (FDKL) method ~~to the SCI/ERA ROM using the plurality of input signals~~.

20. (Original) A method of simulating a fluid flow, comprising:

generating a plurality of statistically independent random numbers for use as input signals; and

performing a singular-value-decomposition directly on a fluid response due to a simultaneous excitation of the plurality of input signals.

21. (Currently Amended) The method of Claim 20, further comprising sampling individual pulse responses for first and second time steps generated by subjecting the dynamic system to first and second pulses to provide Markov parameters for the first and second time steps.

22. (Currently Amended) The method of Claim 20, further comprising, for the system response generated by the simultaneous excitation of the plurality of input signals, sampling the system response y^n for $n = 0, 1, 2, \ldots$ M to provide a Single-Composite-Input (SCI) response for a set of M steps, and defining $\mathbf{H}_{c0}$ and $\mathbf{H}_{c1}$ matrices using the SCI response for the set of M steps as follows:

$$
\begin{aligned}
\mathbf{H}_{c0} &\equiv [\mathbf{y}_{c0}^1 \; \mathbf{y}_{c0}^2 \cdots \mathbf{y}_{c0}^{M-1}] \\
&= \mathbf{C}[\mathbf{x}^1 \; \mathbf{x}^2 \ldots \mathbf{x}^{M-1}] \qquad (25) \\
\mathbf{H}_{c1} &\equiv [\mathbf{y}_{c1}^1 \; \mathbf{y}_{c1}^2 \cdots \mathbf{y}_{c1}^{M-1}] \\
&= \mathbf{C}\mathbf{A}[\mathbf{x}^1 \; \mathbf{x}^2 \ldots \mathbf{x}^{M-1}] \qquad (26)
\end{aligned}
$$

SVD of $\mathbf{H}_{c0}$ yields

$$
\begin{aligned}
\mathbf{H}_{c0} &\equiv \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T \\
&\simeq [\mathbf{U}_n \; \mathbf{U}_z] \begin{bmatrix} \boldsymbol{\Sigma}_n & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_n^T \\ \mathbf{V}_z^T \end{bmatrix} \\
&= \mathbf{U}_n \boldsymbol{\Sigma}_n^{1/2} \boldsymbol{\Sigma}_n^{1/2} \mathbf{V}_n^T \qquad (27)
\end{aligned}
$$

23. (Currently Amended) The method of Claim 22, ~~further obtaining fluid~~ wherein performing a singular-value decomposition includes performing a singular-value decomposition on the matrices $\mathbf{H}_{c0}$ and $\mathbf{H}_{c1}$ to obtain system matrices (A, B, C, D) approximately as follows:

$$
\begin{aligned}
\mathbf{D} &= \mathbf{Y}^0 \qquad (28) \\
\mathbf{C} &\simeq \mathbf{U}_n \boldsymbol{\Sigma}_n^{1/2} \qquad (29) \\
\mathbf{B} &\simeq \boldsymbol{\Sigma}_n^{-1/2} \mathbf{U}_n^T \mathbf{Y}^1 \qquad (30) \\
\mathbf{A} &\simeq \boldsymbol{\Sigma}_n^{-1/2} \mathbf{U}_n^T \mathbf{H}_{c1} \mathbf{V}_n \boldsymbol{\Sigma}_n^{-1/2} \qquad (31)
\end{aligned}
$$

24. (Currently Amended) The method of Claim 22, further comprising, for the system response generated by the simultaneous excitation of the plurality of input signals, sampling the system response y^n for $n = 0, 1, 2, \ldots$ M to provide a Single-Composite-Input (SCI) response for a set of M steps, and defining augmented $\mathbf{H}_{c01}$ and $\mathbf{H}_{c11}$ matrices using the SCI response for the set of M steps as follows:

$$\mathbf{H}_{c01} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^K \end{bmatrix} \left[\mathbf{x}^1 \ \mathbf{x}^2 \ \ldots \ \mathbf{x}^{M-1}\right]$$

$$= \begin{bmatrix} y_{c0}^1 & y_{c0}^2 & \cdots & y_{c0}^{M-1} \\ y_{c1}^1 & y_{c1}^2 & \cdots & y_{c1}^{M-1} \\ \cdots & \cdots & \cdots & \cdots \\ y_{cK}^1 & y_{cK}^2 & \cdots & y_{cK}^{M-1} \end{bmatrix} \tag{32}$$

$$\mathbf{H}_{c11} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^K \end{bmatrix} \mathbf{A}\left[\mathbf{x}^1 \ \mathbf{x}^2 \ \ldots \ \mathbf{x}^{M-1}\right]$$

$$= \begin{bmatrix} y_{c1}^1 & y_{c1}^2 & \cdots & y_{c1}^{M-1} \\ y_{c2}^1 & y_{c2}^2 & \cdots & y_{c2}^{M-1} \\ \cdots & \cdots & \cdots & \cdots \\ y_{cK+1}^1 & y_{cK+1}^2 & \cdots & y_{cK+1}^{M-1} \end{bmatrix} \tag{33}$$

where

$$\begin{aligned} y_{ck}^n &= \mathbf{C}\mathbf{A}^k\mathbf{x}^n \\ &= y^{n+k} - \sum_{i=1}^{N_i} y_i^0 r_i^{n+k} - \sum_{i=1}^{N_i} y_i^1 r_i^{n+k-1} - \\ &\quad \ldots - \sum_{i=1}^{N_i} y_i^k r_i^n \end{aligned}$$

25. (Original) The method of Claim 20, wherein at least some of the input signals are at least one of filtered through a low-pass filter, applied in multiple step inputs in a sequential manner, and applied in multiple pulse inputs in a sequential manner.